

# QCD string as an effective string

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There are two cases where QCD string is described by an effective theory of long strings: the static potential and meson scattering amplitudes in the Regge regime. I show how the former can be solved in the mean-field approximation, justified by the large number of space-time dimensions, and argue that it turns out to be exact for the Nambu–Goto string. By adding extrinsic curvature I demonstrate how the tachyonic instability of the ground-state energy can be cured by operators less relevant in the infrared.

Based on the talks given at the NBI Summer Institute “Strings, gauge theory and the LHC”, Copenhagen August 22 – September 02, 2011 and the Workshop “Low dimensional physics and gauge principles”, Yerevan September 21 – 26, 2011.

Keywords: QCD string, Lüscher term, mean-field approximation, extrinsic curvature, tachyonic instability

## I. INTRODUCTION

As is well known since 1970’s – early 1980’s, QCD string is not fundamental but is rather formed by fluxes of the gluon field at distances larger than the confinement scale. It has the meaning of an *effective* string which makes sense as a string in the limit when it is long. It can be then consistently quantized in  $d = 3 + 1$  dimensions order by order in the inverse length.

In the present talk I shall briefly review this approach and describe how the series in the inverse length can be summed up for a pure bosonic Nambu–Goto string by the mean-field method. From the viewpoint of an effective string it is the most relevant operator in the infrared. I compare the results with recent Monte-Carlo data for the spectrum of QCD string and argue why they are well described by the mean-field approximation. I also incorporate the next relevant operator – the extrinsic curvature – and demonstrate how the tachyonic instability of the ground-state energy can be cured by operators less relevant in the infrared.

## II. QCD STRING AS SUCH

QCD string is formed by fluxes of the gluon field at the distances larger than the confinement scale  $1/\Lambda_{\text{QCD}}$ , where the lines of force between static quarks are collimated into a tube. This picture is supported by numerous Monte-Carlo simulations and agrees with linear hadron Regge trajectories seen in experiment. Perturbative QCD works at small distances (thanks to asymptotic freedom), while an effective string theory works at large distances.

QCD string is not pure bosonic Nambu–Goto string, as was first shown by Migdal and Y.M. (1979) [1] from the loop equation. Extra (fermionic) degrees of freedom are required at the string worldsheet, as advocated by Migdal (1981) [2], to satisfy the loop equation for self-intersecting Wilson loops. But the asymptote of large loops is universal and described by a classical string

$$W(C) \stackrel{\text{large } C}{\propto} e^{-K S_{\min}(C)} \implies \text{the area law} = \text{confinement.} \quad (1)$$

Here  $K$  is the string tension.

Semiclassical fluctuations of a long string were elegantly calculated by Lüscher, Symanzik, Weisz (1980) [3]. For a plane loop the result is given by a conformal anomaly:

$$W(C) \stackrel{\text{plane } C}{\propto} e^{-K S_{\min}(C) + \frac{\#}{24\pi} \int d^2 w \left( \partial_a \ln \left| \frac{dz}{dw} \right| \right)^2}, \quad (2)$$

where the function  $w(z)$  conformally maps the upper half-plane (UHP) onto the interior of the loop. Equation (2) takes a simple form for a  $T \times R$  ( $T \gg R$ ) rectangle

$$W(C) \stackrel{\text{rectangle}}{\propto} e^{-KRT + \frac{\#}{24} \frac{\pi T}{R}} \implies \text{the Lüscher term.} \quad (3)$$

The only unknown constant here is the number  $\#$  of fluctuating degrees of freedom which equal  $d - 2$  for the bosonic string. It agrees with the results of numerical simulations pioneered by Ambjorn, Olesen, Peterson (1984) [4] and De Forcrand, Schierholz, Schneider, Teper (1985) [5].

The  $1/R$  term is *universal* owing to Lüscher's roughening [6] which states that the typical transversal size of the string grows with  $R$  as

$$\langle x_{\perp}^2 \rangle \propto \alpha' \ln(R^2/\alpha') \gg \alpha' \quad \alpha' = 1/2\pi K. \quad (4)$$

Next orders in  $1/R$  are not universal.

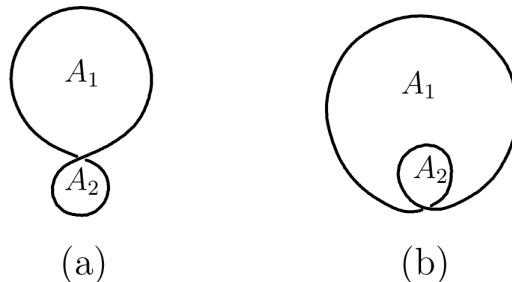


FIG. 1: Loops with one self-intersection:  $A_1$  and  $A_2$  denote the areas of the proper windows.

In the Polyakov (1981) [7] string formulation the Lüscher term (3) can be reproduced *à la* Durhuus, Olesen, Petersen (1984) [8] by conformally mapping UHP onto a  $T \times R$  rectangle.

### III. WHAT TO EXPECT: QCD<sub>2</sub> STRING

QCD<sub>2</sub> is solvable at large  $N$  as demonstrated by 't Hooft (1974) [9]. The interaction vanishes in the axial gauge that immediately results in an exact area law for Wilson loops without self-intersections

$$W(C) = e^{-A(C)} \quad A(C) = \frac{g^2 N}{2} \text{Area.} \quad (5)$$

This looks like bosonic string in  $d = 2$ , but this is not the case for self-intersecting loops as observed by Kazakov, Kostov (1980) [10], Bralić (1980) [11].

The simplest loops with one self-intersection are depicted in Fig. 1. There is nothing special about the loop in Fig. 1a. Equation (5) still holds in this case with  $A(C) = A_1 + A_2$  being the total area:

$$W(\text{a}) = e^{-A_1 - A_2} \quad W(\text{b}) = (1 - 2A_2) e^{-A_1 - 2A_2}. \quad (6)$$

But for the loop in Fig. 1b the exponential of the total (folded) area  $A(C) = A_1 + 2A_2$  is multiplied in Eq. (6) by a nontrivial polynomial which may have negative sign.

The appearance of such preexponential polynomials for the Wilson loops in QCD<sub>2</sub> makes their stringy interpretation more difficult but possible as shown by Gross, Taylor (1993) [12]. These preexponential polynomials (and therefore self-intersections of the loop) become inessential for large loops, so the asymptote (1) is always recovered.

#### IV. EFFECTIVE STRING THEORY

Closed string winding along a compact direction of large radius  $R$  is described by the Polchinski, Strominger (1991) [13] nonpolynomial action

$$S_{\text{eff}} = 2K \int d^2z \partial X \cdot \bar{\partial} X - \frac{\beta}{2\pi} \int d^2z \frac{\partial^2 X \cdot \bar{\partial}^2 X}{\partial X \cdot \bar{\partial} X} + \dots \quad \beta = \frac{26-d}{12}. \quad (7)$$

It can be analyzed order by order in  $1/R$  by expanding about the classical solution

$$X_{\text{cl}}^\mu = (e^\mu z + \bar{e}^\mu \bar{z}) R \quad e \cdot e = \bar{e} \cdot \bar{e} = 0 \quad e \cdot \bar{e} = -1/2. \quad (8)$$

The action (7) emerges from the Polyakov formulation if we integrate over fast fluctuations and express the resulting effective action for slow fields (modulo total derivatives and the constraints) via an induced metric

$$e^{\varphi_{\text{ind}}} = 2 \partial X \cdot \bar{\partial} X \quad (9)$$

(in the conformal gauge), which is not treated independently. This effective string theory has been analyzed [13–15] using the conformal field theory technique order by order in  $1/R$ , revealing the Arvis (1983) [16] spectrum

$$E_n = \sqrt{(KR)^2 + \left(n - \frac{d-2}{24}\right) 8\pi K} \quad 8 \implies 2 \quad \text{for open string} \quad (10)$$

of the Nambu–Goto string in  $d$  dimensions.

The conformal symmetry is maintained in  $d \neq 26$  order by order in  $1/R$ :

$$\delta X^\mu = \epsilon(z) \partial X^\mu - \frac{\beta \alpha'}{4} \partial^2 \epsilon(z) \frac{\bar{\partial} X^\mu}{\partial X \cdot \bar{\partial} X} + \dots + \text{c.c.} \quad (11)$$

It transforms  $X^\mu$  nonlinearly and the corresponding conserved energy-momentum tensor is

$$T_{zz} = -\frac{1}{\alpha'} \partial X \cdot \partial X + \frac{\beta}{2} \frac{\partial^3 X \cdot \bar{\partial} X}{\partial X \cdot \bar{\partial} X} + \mathcal{O}(R^{-2}). \quad (12)$$

Expanding about the classical solution (8):  $X^\mu = X_{\text{cl}}^\mu + Y_{\text{q}}^\mu$ , we obtain

$$T_{zz} = -\frac{2R}{\alpha'} e \cdot \partial Y_{\text{q}} - \frac{1}{\alpha'} \partial Y_{\text{q}} \cdot \partial Y_{\text{q}} - \frac{\beta}{R} \bar{e} \cdot \partial^3 Y_{\text{q}} + \mathcal{O}(R^{-2}). \quad (13)$$

The central charge is determined by the correlator

$$\langle T_{zz}(z_1) T_{zz}(z_2) \rangle = \frac{d+12\beta}{2(z_1 - z_2)^4} + \mathcal{O}((z_1 - z_2)^{-2}) \quad (14)$$

to be  $d+12\beta = 26$  and is cancelled by ghosts at any  $d$ .

## V. MEAN-FIELD APPROXIMATION FOR BOSONIC STRING

The ground state energy of the string determines the static potential between heavy quarks. It was first computed to all orders in  $1/R$  by Alvarez (1981) [17] in the large- $d$  limit, using the saddle point technique in the Nambu–Goto formulation. In the Polyakov formulation (= conformal gauge) these results were extended to any  $d$  by Y.M. (2011) [18] as follows.

Let us consider the (variational) mean-field ansatz with fluctuations included

$$X_{\text{mf}}^1(\omega) = \frac{\omega_1}{\omega_R} R + \delta X^1(\omega) \quad X_{\text{mf}}^2(\omega) = \frac{\omega_2}{\omega_T} T + \delta X^2(\omega) \quad X^\perp(\omega) = \delta X^\perp(\omega) \quad (15)$$

in the world-sheet parametrization, when  $\omega_1, \omega_2 \in \omega_R \times \omega_T$  rectangle. These  $\omega_R, \omega_T$  change under reparametrizations of the loop. The ratio  $\omega_R/\omega_T$  is a variational parameter. It is a reminder of the reparametrization invariance of the boundary for the given parametrization.

The mean-field action with accounting for the Lüscher term reads

$$S_{\text{mf}} = \frac{1}{4\pi\alpha'} \left( R^2 \frac{\omega_T}{\omega_R} + T^2 \frac{\omega_R}{\omega_T} \right) - \frac{\pi(d-2)}{6} \frac{\omega_T}{\omega_R} \quad \frac{1}{6} \Rightarrow \frac{1}{24} \quad \text{for open string.} \quad (16)$$

The minimization with respect to  $\omega_T/\omega_R$  reproduces the square root

$$\left( \frac{\omega_T}{\omega_R} \right)_* = \frac{T}{\sqrt{R^2 - R_c^2}} \quad S_{\text{mf}*} = \frac{T}{2\pi\alpha'} \sqrt{R^2 - R_c^2} \quad R_c^2 = \frac{2\pi^2(d-2)}{3} \alpha'. \quad (17)$$

The singularity at  $R = R_c$  is related to the tachyonic instability as pointed out by Olesen (1985) [19].

For the upper half-plane parametrization, where vertices of the rectangle are at the values  $s_1 < s_2 < s_3 < s_4 \in (-\infty, +\infty)$ , we have

$$\frac{\omega_T}{\omega_R} = \frac{K(\sqrt{r})}{K(\sqrt{1-r})} \quad r = \frac{s_{43}s_{21}}{s_{42}s_{31}} \quad s_{ij} \equiv s_i - s_j \quad (18)$$

as is obtained from the Schwarz–Christoffel mapping. Here  $K$  is the complete elliptic integral of the first kind and the ratio on the right-hand side of Eq. (18) is known as the Grötzsch modulus which is monotonic in  $r$ . Therefore, the minimization with respect to  $r$  gives the same result.

The mean-field approximation is applicable if fluctuations about the minimal surface are small. The ratio of the area of typical surfaces to the minimal area is

$$\frac{\langle \text{Area} \rangle}{A_{\text{min}}} = \frac{1}{RT} \frac{d}{dK} S_{\text{mf}} = \frac{1 - R_c^2/2R^2}{\sqrt{1 - R_c^2/R^2}}, \quad (19)$$

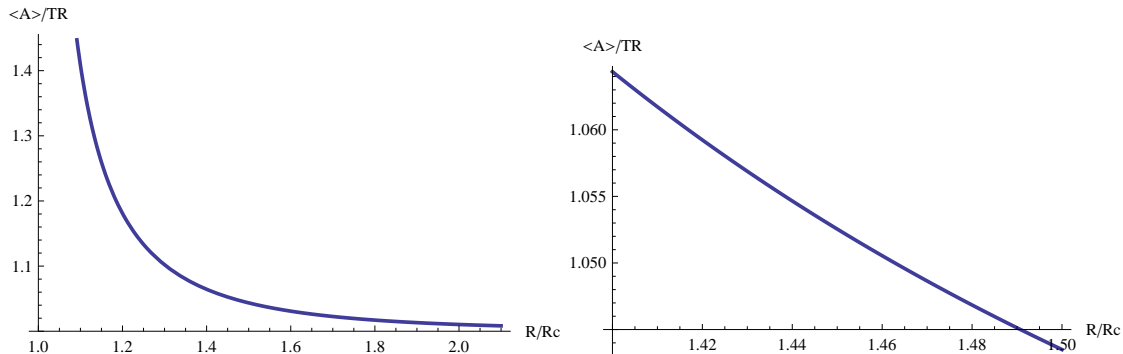


FIG. 2: Plot of the ratio (19). The region near  $R/R_c \approx 1.4$  is magnified in the right figure.

which is plotted in Fig. 2. It approaches 1 for large  $R/R_c$ , implying small fluctuations, but diverges when  $R \rightarrow R_c$  from above, so typical surfaces become very large and the mean-field approximation ceases to be applicable.

The mean-field works generically at large  $d$  but is expected to be exact for the bosonic string at any  $d$ . The arguments are:

1. it is true in the semiclassical approximation;
2. it reproduces an exact result in  $d = 26$ ;
3. it agrees with the existence of a massless bound state in  $N = \infty$  QCD<sub>2</sub> for massless quarks as shown by 't Hooft (1974) [9].

For QCD string we expect that the mean-field approximation works with an exponential accuracy  $\exp(-C R/R_c)$ , as is explained below.

The coefficient of  $d - 2$  in the above formulas is simply the number of fluctuating (transverse) degrees of freedom in the static gauge. In conformal gauge the path integral over reparametrizations<sup>1</sup> of the boundary contributes 24 as is shown by Olesen, Y.M. (2010) [20], ghosts contribute 26, the fluctuations of  $X^\mu$  contribute  $d$ . All together we get  $d + 24 - 26 = d - 2$  again.

## VI. COMPARISON WITH MONTE-CARLO DATA

The stringy spectrum (10) has been recently compared with the results of the very interesting Monte-Carlo computations by Athenodorou, Bringoltz, Teper (2010) [21] of

<sup>1</sup> It is a synonym of the path integrals over boundary metrics or boundary values of the Liouville field in the Polyakov formulation.

the spectrum of closed winding string (flux tube) of circumstance  $R$  in 3+1 dimensional  $SU(N)$  lattice gauge theory. The agreement is absolutely beautiful down to the distances

$$R/R_c \geq 1.4 \quad \sqrt{K}R_c = \sqrt{\frac{d-2}{3}}\pi \approx 1.44. \quad (20)$$

The question immediately arises as to whether or not these results indicate that QCD string is indeed the Nambu–Goto one? To answer this question, let us look at Fig. 2, where it is seen that the ratio (19) is a rather sharp function, approaching infinity as  $R/R_c \rightarrow 1$  from above. Large values of the ratio imply that typical surfaces have large area (in units of  $TR$ ) so that the fluctuations are large.<sup>2</sup> This is of course the case where the mean-field approximation does not work. But for  $R/R_c \geq 1.4$ , where the Monte-Carlo data are available, the ratio is smaller than 1.1 which means that the mean field nicely works and we can restrict ourselves in the effective action of QCD string only by the quadratic operator which is most relevant in the infrared. For the values of  $R/R_c$  closer to 1, other operators will be apparently no longer negligible. We shall explicitly consider in the next section the operator of next relevance in the infrared – the extrinsic curvature.

## VII. QCD STRING AS RIGID STRING

String with extrinsic curvature in the action was introduced in the given context by Polyakov (1986) [22] and Kleinert (1986) [23]. The original idea was that it may provide rigidity of the string that makes stringy fluctuation smoother. We shall momentarily see this is indeed the case after certain subtleties will be resolved, in spite of some contradictory statements in the literature [24–26] (for a review see Ref. [27]).

The action of the bosonic string with the extrinsic curvature term reads

$$S_{\text{rigid string}} = \frac{K}{2} \int d^2\omega \partial_a X \cdot \partial_a X + \frac{1}{2\alpha} \int d^2\omega \frac{1}{\sqrt{g}} \Delta X \cdot \Delta X, \quad (21)$$

where  $\alpha$  is dimensionless constant. It is to be distinguished from intrinsic (or scalar) curvature

$$R = D^2 X \cdot D^2 X - D^a D^b X \cdot D_a D_b X \quad (22)$$

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<sup>2</sup> This crumpling of the surfaces is related to the tachyonic instability and is not expected to happen for QCD string.

that leads to the Gauss–Bonnet term in 2d  $\implies$  the Euler character. The original motivation was that rigidity smoothen crumpling of the surfaces.

Introducing  $\rho = \sqrt{g}$  and the Lagrange multipliers  $\lambda^{ab}$ , we rewrite the action (21) as

$$S_{\text{r.s.}} = K \int d^2\omega \rho + \frac{1}{2\alpha} \int d^2\omega \frac{1}{\rho} \Delta X \cdot \Delta X + \frac{1}{2} \int d^2\omega \lambda^{ab} (\partial_a X \cdot \partial_b X - \rho \delta_{ab}). \quad (23)$$

We consider a mean-field (variational) ansatz, when only  $X^\perp$  fluctuates. It is exact at large  $d$  but approximate at finite  $d$  (like summing bubble graphs for an  $O(d)$ -vector field). We write

$$\begin{aligned} X_{\text{mf}}^1(\omega) &= \frac{\omega_1}{\omega_R} R & X_{\text{mf}}^2(\omega) &= \omega_2 \quad (\omega_T = T) & X^\perp(\omega) &= \delta X^\perp(\omega) \\ \rho_{\text{mf}}(\omega) &= \rho & \lambda_{\text{mf}}^{11}(\omega) &= \lambda^{11} & \lambda_{\text{mf}}^{22}(\omega) &= \lambda^{22} & \lambda_{\text{mf}}^{12}(\omega) &= \lambda_{\text{mf}}^{21}(\omega) = 0 \\ \frac{1}{T} S_{\text{mf}} &= \frac{1}{2} \left( \lambda_{11} \omega_R + \lambda_{22} \frac{R^2}{\omega_R} \right) + \rho \left( K - \frac{\lambda^{11}}{2} - \frac{\lambda^{22}}{2} \right) \omega_R \\ &\quad + \frac{d}{2T} \text{tr} \ln \left( -\lambda^{11} \partial_1^2 - \lambda^{22} \partial_2^2 + \frac{1}{\alpha \rho} (\partial_1^2 + \partial_2^2)^2 \right). \end{aligned} \quad (24)$$

The determinant in the last line equals

$$\frac{d}{2T} \text{tr} \ln(\dots) \longrightarrow \begin{cases} 1) & -\frac{\pi d}{6\omega_R} \sqrt{\frac{\lambda^{22}}{\lambda^{11}}} & \alpha \rightarrow \infty \\ 2) & -\frac{\pi d}{3\omega_R} + \frac{d}{2} \sqrt{\alpha \rho \lambda^{11}} & \alpha \rightarrow 0 \end{cases} \quad (\text{closed string}) \quad (25)$$

as  $\alpha \rightarrow \infty$  or  $\alpha \rightarrow 0$ . Both limiting cases can be analyzed analytically:

- 1) the same mean field as above (large  $\alpha$ ) Alvarez (1981) [17]
- 2) solvable in square roots (small  $\alpha$ ) Polchinski, Yang (1992) [28]

For small  $\alpha$  we have [28]

$$E_0 = \lambda^{11} \omega_R \quad \sqrt{\lambda^{11}} = \frac{3}{8} \frac{d\sqrt{\alpha}}{R} + \sqrt{\frac{9}{16} \frac{d^2 \alpha}{R^2} + K - \frac{\pi d}{3R^2}} \quad \omega_R = \sqrt{R^2 - \frac{dR}{2} \sqrt{\frac{\alpha}{\lambda^{11}}}}. \quad (26)$$

The ground state energy  $E_0$  is plotted versus  $R/R_c$  for various  $\alpha$  in Fig. 3. The tachyonic singularity moves left to smaller values of  $R/R_c$  with decreasing  $\alpha$ , and then returns back to  $R/R_c = 1$  as  $\alpha \rightarrow 0$ .

The lines in Fig. 3 are drawn using exact formulas by Olesen, Yang (1987) [24], Braaten, Pisarski, Tse (1987) [25], Germán, Kleinert (1988) [26]. Integrating in Eq. (24) over  $\mathbf{k}_2$  (as  $T \rightarrow \infty$ ), regularizing via the the zeta function and introducing

$$\Lambda = \frac{\sqrt{\alpha \rho \lambda^{11}} \omega_R}{2\pi} \quad (27)$$



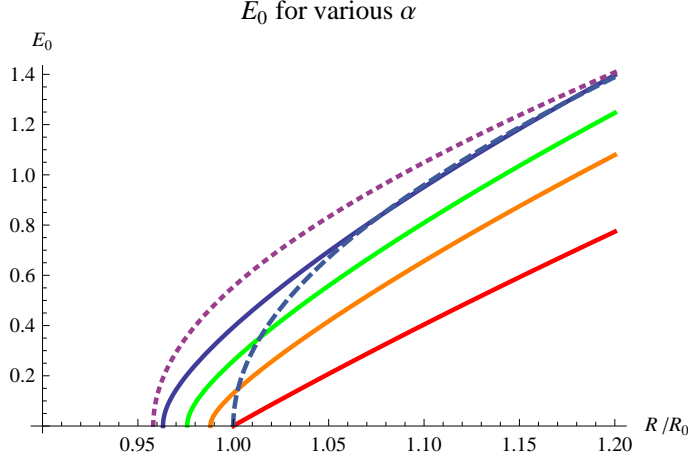


FIG. 3: Ground state energy versus  $R/R_c$  for various  $\alpha$ . The (blue) dashed line emanated from  $R/R_c = 1$  corresponds to  $\alpha = \infty$  (*i.e.* no rigidity = pure Nambu–Goto). The other line corresponds to  $\alpha \sim 10, 0.3, 0.2, 0.1, 0$  from left to right.

instead of  $\rho$ , we find

$$\begin{aligned}
\frac{1}{T} S_{\text{mf}} = & \frac{1}{2} \left( \lambda_{11} \omega_R + \lambda_{22} \frac{R^2}{\omega_R} \right) + \left( \frac{2K}{\lambda^{11}} - 1 - \frac{\lambda^{22}}{\lambda^{11}} \right) \frac{2\pi^2 \Lambda^2}{\alpha \omega_R} \\
& + \frac{2\pi d}{\omega_R} \left[ -\frac{1}{6} + \frac{\Lambda}{2} + \frac{\Lambda^2}{4} \left( 1 + \frac{\lambda^{22}}{\lambda^{11}} \right) \ln \frac{1}{\mu a_{\text{UV}}} \right] \\
& + \frac{2\pi d}{\omega_R} \sum_{n \geq 1} \left[ \sqrt{\frac{\Lambda^2}{2} + n^2 + \Lambda \sqrt{\frac{\Lambda^2}{4} + \left( 1 - \frac{\lambda^{22}}{\lambda^{11}} \right) n^2}} \right. \\
& \left. + \sqrt{\frac{\Lambda^2}{2} + n^2 - \Lambda \sqrt{\frac{\Lambda^2}{4} + \left( 1 - \frac{\lambda^{22}}{\lambda^{11}} \right) n^2}} - 2n - \frac{\Lambda^2}{4n} \left( 1 + \frac{\lambda^{22}}{\lambda^{11}} \right) \right], \quad (28)
\end{aligned}$$

where  $a_{\text{UV}}$  is an ultraviolet cutoff, introduced by

$$\sum_{n \geq 1} \frac{1}{n} = \ln \frac{1}{\mu a_{\text{UV}}}. \quad (29)$$

The renormalization of the parameters  $K$  and  $\alpha$  of the bare action (21) is to be performed in Eq. (28) by introducing (renormalized)

$$\alpha(\mu) = \frac{\alpha}{1 - \frac{\alpha d}{4\pi} \ln \frac{1}{\mu a_{\text{UV}}}} \quad K(\mu) = \frac{K}{1 - \frac{\alpha d}{4\pi} \ln \frac{1}{\mu a_{\text{UV}}}} \quad (30)$$

as is prescribed by asymptotic freedom of the model [22, 23]. Then UV divergences disappear in Eq. (28), so the result is finite.

### VIII. INDUCED EXTRINSIC CURVATURE

A conclusion from the previous section is that adding extrinsic curvature improves the situation but does not cure the problem of tachyonic instability which is not present for QCD string. Thus, more operators of lower dimensions (not relevant in the infrared) have to be added within the effective string theory description of QCD string. They can be systematically induced by internal degrees of freedom of QCD string, *e.g.* massive fermions or higher dimensions, *à la* Sakharov's induced gravity.

The determinant of massless 2d Laplacian (or the Dirac operator squared) in the conformal gauge shows how it may work:

$$\text{tr} \ln \Delta = \frac{1}{12\pi} \int d^2z (\mu_0 e^\varphi \mp \partial\varphi \bar{\partial}\varphi). \quad (31)$$

For the induced metric  $e^\varphi = 2\partial X \cdot \bar{\partial} X$ , we have

$$\int d^2z \partial\varphi \bar{\partial}\varphi = \frac{1}{4} \int d^2z e^{-\varphi} \Delta X \cdot \Delta X \quad (32)$$

and there are no other operators of the same dimension as the extrinsic curvature.

A logarithmically divergent coefficient appears, when the extrinsic curvature is induced by 4d fermions pulled back to the string worldsheet, as calculated by Sedrakian, Stora (1987) [29], Wiegmann (1989) [30], Parthasarathy, Viswanathan (1999) [31]. Likewise, the extrinsic curvature is induced by higher dimensions in the AdS/CFT correspondence with confining background if # of massless modes = # of modes that acquired mass, as demonstrated by Greensite, Olesen (1999) [32].

### IX. CONCLUSION

- QCD string can be viewed as an effective long string and analyzed in  $d = 4$  by the mean-field method.
- Two important applications of this technique:
  - ground state energy of QCD string,
  - meson scattering amplitudes in the Regge regime [18] (not described in this talk).
- Monte Carlo data for the spectrum of QCD string can be well described by only the most relevant operator (Nambu–Goto).
- Extrinsic curvature softens the tachyonic problem and may be induced by additional degrees of freedom of QCD string.

### Acknowledgement

I am indebted to A. Gonzalez-Arroyo, P. Olesen and M. Teper for useful discussions.

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